DEMP21

Partial Differential Equations of Mathematical Physics and Applications

Como, September 13-17, 2021

Main Speakers

Giovanni Paolo Galdi (University of Pittsburgh)

Alain Haraux (Pierre and Marie Curie University - Paris)

Walter Lacarbonara (Sapienza University of Rome)

Invited Speakers

Elvise Berchio (Polytechnic University of Turin)

Denis Bonheure (Free University of Brussels)

Francesca Colasuonno (University of Bologna)

Alberto Ferrero (University of Eastern Piedmont)

Céline Grandmont (INRIA - Paris)

Gabriela Holubová (University of West Bohemia - Pilsen)

Paolo Maremonti (University of Campania)

Andrei Metrikine (Delft University of Technology)

Šárka Nečasová (Czech Academy of Sciences - Prague)

Sebastian Schwarzacher (Charles University - Prague)

Axelle Viré (Delft University of Technology)

Young Speakers

Alessio Falocchi (Polytechnic University of Turin)

Matteo Fogato (Polytechnic University of Milan)

Lorenzo Liverani (Polytechnic University of Milan)

Clara Patriarca (Polytechnic University of Milan)

Gianmarco Sperone (Charles University - Prague)

Elsy Wehbe (University of Pau and Pays De l'Adour)

Poster presenters

Matteo Caggio (Czech Academy of Sciences - Prague)

Antonín Češík (Charles University - Prague)

Erin Maree Ellefsen (University of Colorado Boulder)

Alberto Girelli (University of Milano-Bicocca)

Malte Kampschulte (Charles University - Prague)

Lyndsey Wong (University of Colorado Boulder)

Schedule of the Workshop

	Monday 13th	Tuesday 14th	Wednesday 15th	Thursday 16th	Friday 17th
9:00-9:55	Haraux*	Bonheure	Haraux*	Lacarbonara	Schwarzacher
9:55-10:50	Haraux*	Grandmont	Haraux*	Lacarbonara	Ferrero*
10:50-11:10	*Coffee break*	*Coffee break*	*Coffee break*	*Coffee break*	*Coffee break*
11:10-12:05	Nečasová*	Haraux*	Lacarbonara	Holubová	Lacarbonara
12:05-13:00	Maremonti	Haraux*	Lacarbonara	Haraux*	Lacarbonara
13:00-14:30	*Lunch*	*Lunch*	*Lunch*	*Lunch*	*Lunch*
14:30-15:25	Galdi*	Galdi*		Metrikine	Galdi*
15:25-16:20	Galdi*	Galdi*		Viré*	Galdi*
16:20-16:50	*Poster&Coffee*	*Poster&Coffee*		*Poster&Coffee*	
16:50-17:45	Berchio	Colasuonno		Galdi*	
17:50-18:10	Falocchi	Sperone		Patriarca	
18:10-18:30	Fogato	Liverani		Wehbe*	
19:00-21:30			*Social Dinner*		

*: on-line lecture

Courses

Topics in Mathematical Fluid Mechanics

Giovanni P. GALDI
University of Pittsburgh - U.S.A.

This series of seven lectures will be dedicated to three different fundamental areas of research in mathematical fluid mechanics. More specifically, Lectures 1-3 are dedicated to the flow of a Navier-Stokes liquid past a three-dimensional rigid obstacle; Lectures 4-5 concern the motion of the coupled system constituted by a rigid body with a liquid-filled interior cavity, and, finally, Lectures 6-7 are devoted to the study of self-propulsion of bodies in a Navier-Stokes liquid.

To follow, are the abstracts of the lectures for each of the three topics.

Flow of a Navier-Stokes liquid past an obstacle

We shall perform a mathematical analysis of one of the oldest and most investigated problems in fluid mechanics, namely, the motion of a viscous liquid past a fixed rigid obstacle. Assuming the flow is uniform at large spatial distance (infinity) from the body, We will mainly focus on the determination of the motion characteristics at Reynolds numbers, Re, of arbitrary size. Thus, besides basic issues like existence and regularity, we shall also discuss the occurrence of (multiple) steady and time-periodic bifurcating phenomena, along with a quick (and, unfortunately, to date still incomplete) glance at the behavior in time in the limit of very large Re. Certain significant open questions will be identified and analyzed also in the case when the obstacle is deformable.

Motion of a Rigid Body with a Liquid-Filled Interior Cavity

Problems involving the motion of a rigid body with a cavity filled with a viscous liquid are of fundamental interest in several applied areas of research, including dynamics of flight, space technology, and geophysical problems. Besides its important role in physical and engineering disciplines, the motion of these coupled systems generates a number of mathematical questions, which are intriguing and challenging at the same time. They are principally due to the occurrence of different and coexisting dynamic properties, such as the dissipative nature of the liquid, and the conservative character of some components of the angular momentum of the coupled system as a whole. One important characteristic of this interaction is that the presence of the liquid can dramatically influence the motion of the rigid body and produce a "stabilizing" effect that, in some cases, can even bring the coupled system to rest. Objective of the lectures is to furnish a rather complete mathematical analysis of the dynamics of such systems and provide a rigorous explanation of some of the most relevant observed phenomena.

Self-Propulsion of a Rigid Body in a Viscous Liquid by Time-Periodic Boundary Data

Vibration-induced propulsion of a rigid body is a well studied subject in robotics engineering. In this case, the motion of the body is due by its interaction with a coarse surface. Recently, applied scientists began the study of similar phenomena when the body is embedded in a resistive medium, like a viscous liquid where propulsion is obtained by the oscillation of internal masses producing a time-periodic interaction of the surface of the body with the liquid. The latter can be modeled by a boundary velocity $\mathbf{v}_*$ of a given period. In these lectures we will furnish a rigorous mathematical analysis of this problem with the main objective of finding conditions on $\mathbf{v}_*$ ensuring that the body performs a non-zero net motion, namely, it can cover any given distance in a finite time. We show that the approach to the problem depends on whether the averaged value of $\mathbf{v}_*$ over a period of time is (case (b)) or is not (case (a)) identically zero. In case (a) the problem can be solved in a relatively straightforward way. In case (b), however, the question is much more complicated, since we prove that self-propulsion is a genuinely nonlinear phenomenon. We thus show that the problem can be addressed by a contradiction argument performed directly on the nonlinear system. As a result, we find sufficient conditions for self-propulsion also in case (b), which, by means of counterexamples, turn out to be in general also necessary. Several basic questions remain, however, still open.

The method of adapted energies for second order evolutions with dissipation

Alain HARAUX
Pierre and Marie Curie University
Paris - France

The mini-course will be divided into four lectures, their common feature being the use of adapted energy functions to derive global information on the solutions of second order differential systems.

1) The first two lectures are about Duffing's equation and some related infinite dimensional problems, in the presence of a linear dissipation and a source term. In Lecture 1, we investigate boundedness and stability questions in the so-called single well case, and in Lecture 2 we are mainly concerned with structural stability questions with respect to small exterior forces for the twin-well case (corresponding to one unstable equilibrium and two stable equilibria when the source and the damping term are absent).

2) Lecture 3 is devoted to brutal changes produced by dissipation, more precisely, the study of equations involving a super-linear damping. We examine whether or not the action of the nonlinear damping is able to map instantaneously the whole phase space on a bounded subset as it is the case for parabolic problems. It turns out that for this to happen, we need also the elastic restoring force to be nonlinear, and to be in some sense stronger than the damping. Very precise universal estimates of the solutions are given in that case.

3) Lecture 4 is devoted to the opposite situation of very slow phenomena produced by a weak linear damping. One model for this is the damped Kepler problem, related to a conjecture of Euler who imagined a long term collapse of the solar system as a consequence of the friction of planets against aether. We show that the same model might, if it has some relevance at the level of atoms, explain some strange phenomena appearing in different fields of science and technology: dysfunctioning of some systems involving different materials such as polymeres and metal, the redshift of distant galaxies and the gigantism of insects and plants in the carboniferous era.

Modeling, analysis and control of long-span bridge dynamics

Walter LACARBONARA
Sapienza University of Rome - Italy

This series of lectures will review first the foundational aspects of nonlinear mechanics which drive the modeling of nonlinear structures with a 1D compact support such as cables/strings and beams [1]. In turn, the modeling of arch bridges [2] or suspension bridges [3] will be tackled step-by-step to show how the systems of governing partial-differential equations are obtained.

Subsequently, the analysis aspects will be discussed with respect to the fundamental wind-induced instabilities affecting these slender structures such as torsional divergence [3], buckling, galloping and flutter [4, 6]. These instabilities are first framed in the context of bifurcation theory. One of the powerful asymptotic methods used to study the post-critical solutions, the method of multiple scales, will be reviewed. Then its application to some meaningful problems will be shown to gain insights into the solutions and their variations with the system parameters.

In the final part of the lecture series, application aspects dealing with control of the instabilities (bifurcation control) and control of the post-critical solutions will be discussed. The strategic insertion of nonlinear vibration absorbers in suspension bridge decks will be shown to shift the flutter speed and reduce the limit cycle oscillations amplitude [5].

References

[1] Lacarbonara W. Nonlinear Structural Mechanics. Theory, dynamical phenomena and modeling. Springer, 2013

[2] W. Lacarbonara, A. Arena (2011) Flutter of an arch bridge via a fully nonlinear continuum formulation, *ASCE Journal of Aerospace Engineering* **24**:112–123. doi:10.1061/(ASCE)AS.1943-5525.0000059.

[3] A. Arena, W. Lacarbonara (2012) Nonlinear parametric modeling of suspension bridges under aeroelastic forces: torsional divergence and flutter. *Nonlinear Dynamics* **70**:2487–2510. doi:10.1007/s11071-012-0636-3.

[4] A. Arena, W. Lacarbonara, P. Marzocca (2013) Nonlinear aeroelastic formulation and post-flutter analysis of flexible high-aspect-ratio wings, *AIAA Journal of Aircraft* **50**:1748–1764, doi:hiip://arc.aiaa.org/doi/abs/10.2514/1.C032145.

[5] A. Casalotti, W. Lacarbonara, A. Arena (2014) Mitigation of post-flutter oscillations in suspension bridges by hysteretic tuned mass dampers. *Engineering Structures* **69**: 62–71. doi:10.1016/j.engstruct.2014.03.001.

[6] A. Arena, W. Lacarbonara, P. Marzocca (2016) Post-critical behavior of suspension bridges under nonlinear aerodynamic loading. *Journal of Computational and Nonlinear Dynamics* bf 11(1), 011005 (11 pages), doi:10.1115/1.4030040.

Invited Lectures

Optimization of eigenvalues of partially hinged composite plates and related theoretical issues

Elvise BERCHIO, Polytechnic University of Turin

We consider the spectrum of non-homogeneous partially hinged plates having structural engineering applications. A possible way to prevent instability phenomena is to optimize the frequencies of certain oscillating modes with respect to the density function of the plate; we prove existence of optimal densities and we investigate their qualitative properties. The analysis is carried out by showing fine properties of the involved fourth order operator, such as the validity of the positivity preserving property.
Based on a joint work with Alessio Falocchi.

Equilibrium configuration of a rectangular obstacle immersed in a channel flow

Denis BONHEURE, Free University of Brussels

Fluid flows around an obstacle generate vortices which, in turn, generate lift forces on the obstacle. Therefore, even in a perfectly symmetric framework equilibrium positions may be asymmetric. We show that this is not the case for a Poiseuille flow in an unbounded 2D channel, at least for small Reynolds number and flow rate. We consider both the cases of vertically moving obstacles and obstacles rotating around a fixed pin.
This presentation is based on a joint work with F. Gazzola and P. Galdi.

Nonradial solutions to supercritical problems in an annulus

Francesca COLASUONNO, University of Bologna

In this talk, we will present an existence result for the Dirichlet problem associated to the elliptic equation

$$-\Delta u + u = a(x)|u|^{p-2}u$$

in an annulus of $\mathbb{R}^N$, $N \geqslant 3$. Here $p > 2$ is allowed to be supercritical in the sense of Sobolev embeddings, and $a(x)$ is a positive weight. The equation in consideration is obtained from the stationary version of the Keller-Segel system that models the biological phenomenon of chemotaxis, i.e., the oriented motion of cells towards higher concentrations of a certain chemical substance. For this problem, we will find a new type of positive, axially symmetric solutions. Moreover, in the case where a is constant, we will detect a condition, depending only on the exponent p and on the inner radius of the annulus, that ensures that the solution is nonradial. In this setting, the major difficulty to overcome is the lack of compactness in a nonradial framework. The proof of the result relies on a combination of variational methods and dynamical systems techniques.
This talk is based on a joint paper with Alberto Boscaggin, Benedetta Noris, and Tobias Weth.

An orthotropic plate model for decks of suspension bridges

Alberto FERRERO, University of Eastern Piedmont

We compare two different kinds of model for the deck of a suspension bridge, the first one consisting of a single equation coming from the theory of isotropic plates and the second one consisting of a system of two equations, one for vertical deflections and one for torsional deformations. In our discussion, we observe that the model of the system has more degrees of freedom if compared with the one of the isotropic plate. For this reason the system model is supposed to be more appropiate to describe the behavior of the deck of a realistic suspension bridge. A possible strategy to make the plate model efficiency comparable with the system model, can be realized by relaxing the isotropy assumption with a more general condition of orthotropy, which is expected to increase the degrees of freedom in view of the larger number of elastic constants. We formulate this model of orthotropic plate and we compare its behaviour with the one of the system.

Global existence of weak solutions for a fluid beam interaction problem encompassing possible contact

Céline GRANDMONT, INRIA - Paris

We consider a coupled system of PDEs modelling the interaction between a two-dimensional incompressible viscous fluid and a one–dimensional elastic beam located on the upper part of the fluid domain boundary. Starting from the system with additional viscosity on the structure for which no contact occurs between the elastic beam and the bottom of the fluid cavity, we prove the existence of weak solution regarless the possible contact letting this additional viscosity vanishes. To do so we first need to design an appropriate functional framework to define weak solutions in case of contact and to prove compactness result on the sequence of approximated fluid and structure velocities.

From suspension bridges to new threshold values for inverse positivity

Gabriela HOLUBOVÁ, University of West Bohemia - Pilsen

We consider a modified version of a suspension bridge model with a spatially variable stiffness parameter that better reflects the naturally discrete distribution of the bridge hangers. We show that this modification improves the behavior of the model while not changing its qualitative properties, mainly the occurrence of bifurcation phenomena. However, the implementation of the bifurcation theory relies on the existence of a positive stationary solution under positive constant loading.
This brings us to the study of sufficient conditions for the (strict) inverse positivity of the linear fourth order ordinary differential operator with a spatially variable coefficient. We revive more than half a century old tools introduced by J. Schröder and show that the extrema of the variable coefficient can significantly exceed the originally derived bounds.

IBVP in exterior domains of the 2D Stokes problem: a result of non uniqueness in L^∞

Paolo MAREMONTI, University of Campania

We consider the initial boundary value problem in $L^\infty((0,T)\times\Omega)$, also shortly called the *maximum modulus theorem*, for the 2D Stokes system:

$$v_t + \nabla\pi = \Delta v\,,\ \ \nabla\cdot v = 0\,,\ \text{in } (0,T)\times\Omega\,, \tag{1}$$

$\Omega\subset\mathbb{R}^2$ smooth exterior domain, under the assumptions

$$v = 0 \text{ on } (0,T)\times\partial\Omega\,,\ \ v = v_0 \text{ on } \{0\}\times\Omega\,, \tag{2}$$

where the initial datum $v_0\in L^\infty(\Omega)$ is divergence free [1]. That is, we look for a constant c independent of v_0 and a pair (v,π) solution to (1) such that for all $t>0$

$$\begin{gathered} t\|v_t\|_\infty + \big(\tfrac{t}{t+1}\big)\big[\|\Delta v\|_\infty + \|\nabla\pi\|_\infty\big] + \big(\tfrac{t}{t+1}\big)^{\frac12}\|\nabla v\|_\infty + \|v\|_\infty \leqslant c\|v_0\|_\infty\,,\\ \text{and } \lim_{t\to0}(v(t),\varphi) = (v_0,\varphi)\,,\ \ \text{for all } \varphi\in\mathscr{C}_0(\Omega)\,. \end{gathered} \tag{3}$$

We prove that *a priori* several solutions correspond to $v_0\in L^\infty(\Omega)$, divergence free, all satisfying (3).

Stability of structures in interaction with translating continua: PDEs or phenomenological models?

Andrei METRIKINE, Delft University of Technology

Three challenging engineering problems of contemporary civil engineering are addressed in this presentation. The first problem is the stability of a high-speed railway vehicle. This problem is relevant for the development of the hyperloop transportation system and is interesting academically in view of the role of the anomalous Doppler waves that can be radiated into the rail by the vehicle. The second problem is the stability of a deep-water riser in marine currents. This is relevant for the rapidly developing deep water mining technologies and is associated with the vortex-induced vibrations of pipes conveying fluid. The third problem is the ice-induced vibrations of offshore wind turbines. While not well-known, this problem is a beautiful example of self-excited vibrations, in which the failure of ice is synchronized both temporarily and spatially by a vibrating structure.
These three problems, although dissimilar in terms of the underlying physical processes, are unified by a fundamental similarity associated with the fact that the instability is caused by a translating continuum (current or ice) or a moving load (vehicle). Furthermore, the mathematical description of the continuum dynamics and, especially, of the interaction between the structure and the continuum is rather challenging. Therefore, a question is addressed in this presentation as to the optimal ways of the mathematical modelling of the considered engineering problems.

[1] That is $(v_0,\nabla\phi)=0$ for all $\phi\in C_0^1(\overline{\Omega})$.

On the problem of the motion of a rigid body in a compressible fluid

Šárka NEČASOVÁ, Czech Academy of Sciences - Prague

We study a 3D nonlinear moving boundary fluid-structure interaction problem describing the interaction of the fluid flow with a rigid body. The fluid flow is governed by 3D compressible Navier-Stokes equations, while the motion of the rigid body is described by a system of ordinary differential equations called Euler equations for the rigid body. The equations are fully coupled via dynamical and kinematic coupling conditions. We consider two different kinds of kinematic coupling conditions: no-slip and slip. First we consider the Navier-slip boundary condition at the interface as well as at the boundary of the domain and we show existence of a weak solution of the fluid-structure system up to collision, see [3]. Secondly, we show the weak-strong uniqueness principle in the case of no-slip boundary conditions [2]. Finally, we show that as the size of the object converges to zero the system fluid plus rigid body converges to the compressible Navier-Stokes system under some mild lower bound on the mass and the inertia momentum, see [1].

References

[1] Marco Bravin, Šárka Nečasová: *On the vanishing rigid body problem in a viscous compressible fluid*, arXiv:2011.05040

[2] Ondřej Kreml, Šárka Nečasová, Tomasz Piasecki: *Weak-strong uniqueness for the compressible fluid-rigid body interaction*, J. Differential Equations 268 (2020), no. 8, 4756–4785

[3] Šárka Nečasová, Ramaswamy, A. Roy and A. Schlömerkemper: *Motion of a rigid body in a compressible Fluid with Navier-slip boundary condition*, arXiv:2103.08762

A variational approach to fluid-structure interactions

Sebastian SCHWARZACHER, Charles University - Prague

In this talk some recent existence results in fluid-structure interactions are discussed. It focus on the case when visco-elastic bulk solids are interacting with fluids. One of the characteristic difficulties of the respective PDE systems is the variable-in-time fluid domain being a part of the solution.
The construction of solutions is by step-wise minimization. Such a variational approximation seems to be irreplaceable for large deformation solids, since the respective state spaces are (for physical reasons) non-convex. We introduce a two time-scale approximation scheme that is capable to construct weak solutions describing bulk solids interacting with fluids governed by the incompressible or compressible Navier Stokes equations.
The talk is based on collaborations with Benesova, Breit and Kampschulte.

Numerical modelling of fluid-structure interactions for the next-generation of wind turbines

Axelle VIRÉ, Delft University of Technology

Fluid-structure interactions are ubiquitous in science and engineering. Computational fluid dynamics (CFD) models that solve the Navier-Stokes equations can be coupled to a structural dynamics model to simulate such interactions, whilst accounting for non-linearities. This lecture will present some of the algorithms that have been developed to achieve this goal. Their applications to problems of relevance to wind energy will be presented, including the dynamics of vortex-induced vibrations for wind turbine towers, the response of a floating support structure for wind energy harvesting in deep sea, and the deformation of a flexible membrane for airborne wind energy.

Short Talks

Regularity for the 3D evolution Navier-Stokes equations under Navier boundary conditions in some Lipschitz domains

Alessio FALOCCHI, Polytechnic University of Turin

For the evolution Navier-Stokes equations in bounded 3D domains, it is well-known that the uniqueness of a solution is related to the existence of a regular solution. They may be obtained under suitable assumptions on the data and smoothness assumptions on the domain (at least $C^{2,1}$). With a symmetrization technique, we prove these results in the case of Navier boundary conditions in a wide class of merely *Lipschitz domains* of physical interest, that we call *sectors*. Based on a joint work with Filippo Gazzola, Politecnico di Milano.

Asymptotic finite-dimensional approximations for a class of extensible elastic systems

Matteo FOGATO, Polytechnic University of Milan

We consider the equation

$$u_{tt} + \delta u_t + A^2 u + \|A^{\theta/2}u\|^2 A^\theta u = g$$

where A^2 is a diagonal, self-adjoint and positive-definite operator and $\theta \in [0,1]$ and we study some finite-dimensional approximations of the problem. First, we analyze the dynamics in the case when the forcing term g is a combination of a finite number of modes. Next, we estimate the error we commit by neglecting the modes larger than a given N. We then prove, for a particular class of forcing terms, a theoretical result allowing to study the distribution of the energy among the modes and, with this background, we refine the results. Some generalizations and applications to the study of the stability of suspension bridges are given.

On the Moore-Gibson-Thompson equation with memory with nonconvex kernels

Lorenzo LIVERANI, Polytechnic University of Milan

We consider the MGT equation with memory

$$\partial_{ttt}u + \alpha\partial_{tt}u - \beta\Delta\partial_t u - \gamma\Delta u + \int_0^t g(s)\Delta u(t-s)ds = 0.$$

We prove an existence and uniqueness result removing the convexity assumption on the convolution kernel g, usually adopted in the literature. In the subcritical case $\alpha\beta > \gamma$, we establish the exponential decay of the energy, without leaning on the classical differential inequality involving g and its derivative g', namely,

$$g' + \delta g \leqslant 0, \quad \delta > 0,$$

but only asking that g vanishes exponentially fast. This is a joint work with Vittorino Pata and Monica Conti.

An explicit threshold for the appearance of lift on the deck of a bridge

Clara PATRIARCA, Polytechnic University of Milan

We set up the analytical framework for studying the threshold for the appearance of a *lift force* exerted by a viscous steady fluid (the wind) on the deck of a bridge. We model this interaction as in a wind tunnel experiment, where at the inlet and outlet sections the velocity field of the fluid has a *Poiseuille flow* profile. Since in a symmetric configuration the appearance of lift forces is a consequence of non-uniqueness of solutions, we compute an explicit threshold on the incoming flow ensuring uniqueness. This requires building an explicit solenoidal extension of the prescribed Poiseuille flow and bounding some embedding and cutoff constants. This is a joint work with Professor Filippo Gazzola.

References

[1] F. Gazzola, C. Patriarca. An explicit threshold for the appearance of lift on the deck of a bridge, preprint. 2020

[2] F. Gazzola, G. Sperone, Steady Navier-Stokes equations in planar domains with obstacle and explicit bounds for unique solvability. *Archive for Rational Mechanics and Analysis 238(3):1283-1347*, 2020.

Further remarks on radial symmetry and monotonicity for solutions of semilinear higher order elliptic equations

Gianmarco SPERONE, Charles University - Prague

Fifty years after the pubblication of Serrin's pioneering paper on symmetry problems arising in potential theory, we consider semilinear polyharmonic equations under Dirichlet boundary conditions in the unit ball of $\mathbb{R}^n$. After discussing radial properties (symmetry and monotonicity) of positive solutions of such equations, we show that, in conformal dimensions $n = 2m$ with $m \geqslant 1$, the associated Green function satisfies an elegant reflection property involving the inversion in the sphere and the Kelvin transform. This then yields an alternative formula for computing the partial derivatives of the solutions of the polyharmonic problems considered. We also revise a counterexample by Sweers where radial monotonicity fails: by appropriately modifying the source f we numerically obtain radially symmetric and strictly decreasing solutions of the biharmonic equation in the unit ball of $\mathbb{R}^4$ under Dirichlet boundary conditions. This is a joint work with Filippo Gazzola (Politecnico di Milano).

Existence and regularity of a magnetohydrodynamic system with Navier-type boundary conditions in 2-D

Elsy WEHBE, University of Pau and Pays De l'Adour

Magnetohydrodynamic (MHD) is the discipline studying the behaviour of conductive fluids of electricity when their movement is coupled to the electromagnetic field. Here we study in Ω, a multi-connected two dimensional domain, the existence of solutions for a MHD coupling an equation of polymer aqueous solution with Maxwell equation of electromagnetic. These equations are presented, in the stationary case, as the following:

$$\begin{aligned}
&-\nu\Delta\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)(\boldsymbol{u}-\alpha\Delta\boldsymbol{u}) + \nabla\pi - (\boldsymbol{B}\cdot\nabla)\boldsymbol{B} + \frac{1}{2}\nabla(|\boldsymbol{B}|^2) = \mathbf{f} && \text{in } \Omega,\\
&-\Delta\boldsymbol{B} - (\boldsymbol{B}\cdot\nabla)\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{B} + \nabla\theta = 0 && \text{in } \Omega,\\
&\text{div } \boldsymbol{u} = 0, \quad \text{div } \boldsymbol{B} = 0 && \text{in } \Omega,
\end{aligned}$$

where $\boldsymbol{u}$ and $\boldsymbol{B}$ are the velocity field and the magnetic field, π is the pressure of the fluid, θ is an unknown function related to the motion of heavy ions and $\mathbf{f}$ is the external force acting on the fluid. We study the existence of solutions $(\boldsymbol{u},\boldsymbol{B},\pi,\theta)$ in $\boldsymbol{H}^2(\Omega)\times\boldsymbol{H}^2(\Omega)\times\boldsymbol{L}^2(\Omega)\times\boldsymbol{H}^1(\Omega)$ with the Navier-type boundary conditions:

$$\begin{aligned}
\boldsymbol{u}\cdot\boldsymbol{n} = 0, \qquad \text{curl } \boldsymbol{u} = 0, \qquad \text{on } \partial\Omega\\
\boldsymbol{B}\cdot\boldsymbol{n} = 0, \qquad \text{curl } \boldsymbol{B} = 0, \qquad \text{on } \partial\Omega.
\end{aligned}$$

To solve our problem we need some estimations related to the Stokes associated problem. One of the difficulties is the geometry of the domain, supposed here non simply connected. On the other hand, it is shown an additional regularity in $\mathbf{W}^{2,p}(\Omega)$ for the magnetic field. This is a joint work with Cherif Amrouche.

Posters

Singular limits in fluid mechanics: low Mach number flows and dimension reduction

Matteo CAGGIO, Czech Academy of Sciences - Prague

We consider the compressible Navier-Stokes system describing the motion of a viscous fluid confined to a straight layer $\Omega_\delta = (0, \delta) \times \mathbb{R}^2$. We show that the weak solutions in the 3D domain converge strongly to the solution of the 2D incompressible Navier-Stokes equations (Euler equations) when the Mach number tends to zero as well as $\delta \to 0$ (and the viscosity goes to zero). Similarly, we also consider the compressible Euler system describing the motion of an ideal fluid. In the framework of *dissipative measure-valued solutions*, we show the convergence to the strong solution of the 2D incompressible Euler system when the Mach number tends to zero and $\delta \to 0$.

References

[1] Caggio M., Donatelli D., Nečasová Š. and Sun Y., Low Mach number limit on thin domains, Nonlinearity, 33(2), 840-863, 2020.

[2] Caggio M., Ducomet B., Nečasová Š. and Tang T., Low Mach and thin domain limit for the compressible Euler system, Annali di Matematica Pura ed Applicata, 200, 1469-1486, 2021.

Energy estimates in a variational approach to hyperbolic evolutions

Antonín ČEŠÍK, Charles University - Prague

In a recent work, B. Benešová, M. Kampschulte and S. Schwarzacher introduced a variational time-stepping method for solving (non-linear) hyperbolic problems with viscoelastic solids. It is an extension of De Giorgi's famous minimizing movements, using two time scales: the velocity scale τ and the (larger) acceleration scale h. In their method, they first pass with $\tau \to 0$ to obtain energy estimates which then allow to pass with $h \to 0$. In the current work, we dive deeper into the energy estimates to see how much smaller τ needs to be, to allow for a simultaneous limit passage of the two parameters to zero. This is a joint work with Sebastian Schwarzacher.

Efficiently finding equilibrium solutions of nonlocal territorial models in Ecology

Erin Maree ELLEFSEN, University of Colorado Boulder

Nonlocal models can be very useful to describe some phenomena in Ecology. For example, observations of social groups of meerkats suggest nonlocal forces are taken into account when territories develop. However, nonlocal models also pose both analytical and computational challenges. We investigate territory development of meerkats by studying a system of nonlocal continuum equations. We perform a long-wave approximation of this system to investigate a local approximation, and we take advantage of the gradient-flow structure of the local and nonlocal systems in order to find equilibrium solutions. We compare these equilibrium solutions to determine if the local approximation is an appropriate substitute to the nonlocal model for future work. Finally, we utilize spectral methods to find equilibrium solutions more efficiently which allows us to consider a larger system of equations. We consider this approach as we aim to incorporate data into our model.

Mathematical model of a lymph node

Alberto GIRELLI, University of Milano-Bicocca

Lymph nodes are organs scattered all across the lymphatic network and their function is to filter the lymph and break down bacteria, viruses, and waste; these substances are transported inside the lymph nodes thanks to the lymph. Despite its importance, few models that try to describe the behavior of the lymph from a mechanical point of view. My research aims to try to model the behavior of the lymph inside the lymph node and to describe the transport of particles due to the lymph.

Existence through collisions in nonlinear viscoelastodynamics

Malte KAMPSCHULTE, Charles University - Prague

In the real world, collisions between elastic objects are a common sight and intuitively well understood. Mathematically however the situation has been far less clear. While physically a collision is just another example of a force balance, on the level of a Lagrangian continuum mechanical description, it manifests as a temporary non-local boundary condition for the PDE, whose time and place of occurrence depends highly on the solution itself. In the static and quasi-static regime, corresponding existence results have been shown through variational descriptions of the problem. But for the fully dynamical case, collisions have so far either ignored or handled using a simplified model, e.g. repulsive terms. In contrast to this, using a newly developed variational technique for general PDEs of similar dynamic type, we are able to present an existence result for solutions to a physically correct problem involving collisions. Furthermore we are able to arrive at the correct model using only the energy balance and an assumption of matter non-interpenetration. This is joint work with Antonín Češík and Giovanni Gravina.

A mathematical model of wealth distribution through an amenities-based theory

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Gentrification refers to the influx of income into a community leading to the improvement of an area through renovation or the introduction of local amenities. This is usually accompanied by an increase in the cost of living, which displaces lower income populations. To better understand this problem, we will introduce a PDE model for the dynamics of wealth based on amenities. In order to find when we have inhomogeneous solutions to this model, we present two approaches. The first is to perform a linear stability analysis in order to find when small perturbations of constant equilibrium solutions become unstable. The second is to prove the existence of a global bifurcation of these solutions from the constant equilibrium solution.